A Geršgorin Theory for Robust Microgrid Stability Analysis

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***Abstract*—A robust stability assessment approach is presented to efficiently estimate eigenvalues in microgrids in the presence of bounded uncertainties. Through this method, all possible locations of eigenvalues can be directly obtained, which makes repeatedly eigenvalue calculation unnecessary when dealing with uncertainties. More importantly, a quasi-diagonalization technique is established to reduce the conservativeness of the Geršgorin theory. Extensive tests show that the new method enables highly efficient analysis on impact levels of disturbances and offers a useful tool for droop coefficients design which facilitates microgrids stable operation. Besides, test results show that the Geršgorin theory based approach can be effectively combined with other methods to obtain more accurate solutions. These salient features make the new method a powerful tool for planning, operating, and designing future microgrids.**

***Index Terms*—microgrid; robust stability analysis; Geršgorin theory; eigen-analysis; uncertainty; distributed energy resources (DERs)**

I. Introduction

Microgrid is a promising paradigm to enhance power supply resiliency for customers [1]. Nowadays, most power-electronic-interfaced microgrids have very low inertia, making them highly sensitive to disturbances such as intermittent generations from PV or wind [2]. Because microgrid is inevitably subject to small disturbances such as intermittent renewable generation (coupled with load variations), any microgrid that is unstable in terms of small perturbations cannot sustain for long term operations. Quantifying the impact of various uncertainties (disturbances) on microgrid small signal stability is fundamentally important for microgrid planning, operation and control design. Therefore, a principal task in microgrid analysis is to investigate the small signal stability of power-electronic-dominated microgrids under uncertain inputs or outputs, especially under multiple uncertainties (usually 'unknown but bounded' uncertainties characterized by sets).

There exist two major categories of methods to assess small signal stability of microgrids, namely, exact computational approaches such as QR method and perturbation-based methods such as matrix perturbation theory [3]. Both are point-based approaches. When a microgrid is subject to various disturbances, the former methods need to solve eigenvalue problems caused by disturbances one by one, which is extremely tedious and time-consuming. Furthermore, it is difficult to use these methods to quantify and compare the impact of different disturbances on microgrid stability [4]. On the other hand, the latter methods aim at obtaining eigenvalues without repeated calculations when considering disturbances, and the basic idea is to discover the impacts of perturbed parameters on system stability through perturbation analysis. Because these methods are still point-based analysis, they are unable to deal with set-based disturbances.

To overcome the limitations of existing technologies, a novel approach based on Geršgorin theory [5], [6] is developed to efficiently assess the small signal stability of mcirogrids under uncertainties. A salient feature of this approach is the capability of demonstrating and describing variation regions of eigenvalues which reflect a system's small signal stability feature under different disturbances. The major contributions of this new method include: (*i*) It is an on-the-fly solution that directly obtains the location of eigenvalues for a microgrid subject to disturbances, rather than repeatedly solving the eigenvalue problem of the microgrid with on-going disturbances; (*ii*) The detailed Gergorin disks information sheds light on how different disturbances impact microgrid stability, which can be used to pinpoint critical disturbances; (*iii*) It can discover how to change microgrid parameters so as to shift critical eigenvalues into designed or desirable region, which can be utilized to design inverter controller parameters to effectively enhance microgrids stability. Moreover, the new Geršgorin method can be combined with other stability analysis techniques to significantly enhance their performances. For instance, since the Geršgorin approach can divide eigenvalues into different groups, QR method or perturbation theory can then be applied in each specific group exclusively to get more accurate locations of eigenvalues in each group.

The remainder of this paper is organized as follows. Section II introduces the Geršgorin theorem for eigenvalue estimation under system disturbances. Section III describes the Geršgorin analysis of a microgrid system. Besides rigorous theoretical analysis, procedures to get Gergorin disks for a microgrid are provided as well. Numerical tests are provided in Section IV which verify the feasibility and effectiveness of the presented approach. Conclusions are drawn in Section V.

II. Geršgorin Theorem

Geršgorin Theorem is a powerful method for the eigenvalues estimation of dynamical systems. Considering the nonsingular

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finite-dimensional state matrix of a system $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$, the eigenvalue problem is described as follows [7].

$$\begin{cases} \mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i \\ \mathbf{A}^{\mathrm{T}}\mathbf{u}_i = \lambda_i \mathbf{u}_i \end{cases} \tag{1}$$

where λ_i is the i^{th} generalized eigenvalue of the system; $\mathbf{v}_i$ and $\mathbf{u}_i^{\mathrm{T}}$ are the i^{th} right and left eigenvector, respectively, satisfying the orthogonal normalization conditions as shown in (2).

$$\begin{cases} \mathbf{u}_i^{\mathrm{T}}\mathbf{v}_j = \delta_{ij} \\ \mathbf{u}_i^{\mathrm{T}}\mathbf{A}\mathbf{v}_j = \delta_{ij}\lambda_i \end{cases} \tag{2}$$

where δ_{ij} is the Kronecker sign.

Theorem 1. *For any nonsingular finite-dimensional matrix* $\mathbf{A}$ *with* λ_i *as its* i^{th} *eigenvalue, there is a positive integer* k *in N=1,2,...,n such that,*

$$|\lambda_i - a_{kk}| \le r_k(\mathbf{A}) \tag{3}$$

where $r_k(\mathbf{A}) \doteq \sum_{j\in N\setminus\{k\}} |a_{kj}|$. *If* $\sigma(\mathbf{A})$ *denotes a set of all eigenvalues of* $\mathbf{A}$*, then* $\sigma(\mathbf{A})$ *satisfies the following condition*

$$\sigma(\mathbf{A}) \subseteq \Gamma(\mathbf{A}) \doteq \bigcup\nolimits_{k=1}^{n} \Gamma_k(\mathbf{A}) \tag{4}$$

where $\Gamma(\mathbf{A})$ *is the Geršgorin set of nonsingular matrix* $\mathbf{A}$*;* $\Gamma_k(\mathbf{A})$ *is the* k^{th} *Geršgorin disk, and can be expressed as* $\Gamma_k(\mathbf{A}) \doteq \{|x - a_{kk}| \le r_k(\mathbf{A}), x \in \mathbb{R}\}$.

Further details can be found in [5], [6].

III. Geršgorin Analysis of Microgrids under Disturbances

This section develops an enhanced Geršgorin method that significantly improves the accuracy of eigenvalue estimation for microgrids.

A microgrid consisting of DERs, inverters, loads, and network can be expressed by state and algebraic equations [7]. Mathematically, such a microgrid can be described by a set of differential and algebraic equations (DAEs).

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{y}, \mathbf{p}) \\ \mathbf{0} = \mathbf{G}(\mathbf{x}, \mathbf{y}, \mathbf{p}) \end{cases} \tag{5}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state variable vector; $\mathbf{y} \in \mathbb{R}^m$ is the algebraic variable vector; $\mathbf{p} \in \mathbb{R}^p$ is disturbance vector. Linearizing the microgrid system at the initial operation point $(\mathbf{x_0}, \mathbf{y_0})$, one can obtain the following equations when the partial derivative matrix of algebraic equations with respect to algebraic variables $\mathbf{G_y}$ is nonsingular.

$$\Delta\dot{\mathbf{x}} = [\mathbf{F_x} - \mathbf{F_y}\mathbf{G_y}^{-1}\mathbf{G_x}]\Delta\mathbf{x} \tag{6}$$

where $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x_0}$, $\mathbf{F_x}(\mathbf{F_y})$ is the partial derivative matrix of differential equations with respect to state (algebraic) variables, $\mathbf{G_x}$ is the partial derivative matrix of algebraic equations with respect to state variables. The small signal stability feature of a microgrid is governed by the eigenvalues of its state matrix:

$$\mathbf{A}_{MG} = \mathbf{F_x} - \mathbf{F_y}\mathbf{G_y}^{-1}\mathbf{G_x} \tag{7}$$

where $\mathbf{A}_{MG}$ is equivalent to $\mathbf{A}$ in Theorem 1.

A. Geršgorin Disk and Set Calculation

After obtaining the system state matrix, the Geršgorin disk and set can be calculated based on Theorem 1. However, the estimation result of eigenvalue distribution is usually over-approximated, when the state matrix is not strongly diagonally dominant. According to Theorem 1, the distribution of eigenvalue (area of Geršgorin disk) is mainly determined by the non-diagonal elements of state matrix. The more diagonally dominant a state matrix, the smaller its Geršgorin disk, i.e., the estimation accuracy of eigenvalue distribution will be highly improved. Therefore, a quasi-diagonalization technique is established below to reduce the conservativeness of Geršgorin theory.

Taking into account the orthogonal normalization conditions shown in (2), the state matrix $\mathbf{A}_{MG}$ under system disturbances can be quasi-diagonalized as follows.

$$\mathbf{U}_0^{\mathrm{T}}\mathbf{A}_{MG}\mathbf{V}_0 = \mathbf{U}_0^{\mathrm{T}}\mathbf{A}_{MG,0}\mathbf{V}_0 + \mathbf{U}_0^{\mathrm{T}}\mathbf{A}_{MG,D}\mathbf{V}_0 = \mathbf{S}_0 + \mathbf{S}_D \tag{8}$$

where $\mathbf{A}_{MG,0}$ is system state matrix at $(\mathbf{x}_0, \mathbf{y}_0)$; $\mathbf{S}_0$, $\mathbf{U}_0^{\mathrm{T}}$ and $\mathbf{V}_0$ are the corresponding eigenvalue matrix, left eigenvector matrix, and right eigenvector matrix at $(\mathbf{x}_0, \mathbf{y}_0)$, respectively; $\mathbf{A}_{MG,D}$ is the increment of state matrix under disturbances, which is constructed based on a bounded set of uncertainties and will be analyzed in next subsection; $\mathbf{S}_D$ is the increment of eigenvalue matrix. Thus, the eigenvalue problem of a disturbed system is transformed to the analysis of the matrix $\mathbf{S}_D$. And the following expression can be obtained.

$$\Gamma_k(\mathbf{S}_D) = \{|x - s_{kk}| \le r_k(\mathbf{S}_D), x \in \mathbb{R}\} \tag{9}$$

$$\sigma_k(\mathbf{S}_D) \subseteq \Gamma(\mathbf{S}_D) \doteq \bigcup\nolimits_{k=1}^{n} \Gamma_k(\mathbf{S}_D) \tag{10}$$

Therefore, the distribution of each eigenvalue in a system under uncertainties can be expressed as a Geršgorin disk with $\mathbf{S}_0$ as its center and $\Gamma_k(\mathbf{S}_D)$ as its corresponding area.

B. Disturbances Analysis in Microgrids

To help process the wide variety of disturbances in microgrids, those uncertainties are divided into groups, i.e., fluctuations from DERs, changes of loads, perturbations from controllers parameters, disturbances of power exchange at the point of common coupling (PCC), etc. Since the constitutions of $\mathbf{A}_{MG}$ will change accordingly when the system is under disturbances, they can be generally expressed as follows.

$$\mathbf{F_x} = \sum_{i=1}^{N_G} \mathbf{F}_{\mathbf{x},G_i} + \sum_{j=1}^{N_L} \mathbf{F}_{\mathbf{x},L_j} + \sum_{k=1}^{N_P} \mathbf{F}_{\mathbf{x},P_k} + \mathbf{F}_{\mathbf{x},E} + \mathbf{F}_{\mathbf{x},C} \tag{11}$$

$$\mathbf{F_y} = \sum_{i=1}^{N_G} \mathbf{F}_{\mathbf{y},G_i} + \sum_{j=1}^{N_L} \mathbf{F}_{\mathbf{y},L_j} + \sum_{k=1}^{N_P} \mathbf{F}_{\mathbf{y},P_k} + \mathbf{F}_{\mathbf{y},E} + \mathbf{F}_{\mathbf{y},C} \tag{12}$$

$$\mathbf{G_x} = \sum_{i=1}^{N_G} \mathbf{G}_{\mathbf{x},G_i} + \sum_{j=1}^{N_L} \mathbf{G}_{\mathbf{x},L_j} + \sum_{k=1}^{N_P} \mathbf{G}_{\mathbf{x},P_k} + \mathbf{G}_{\mathbf{x},E} + \mathbf{G}_{\mathbf{x},C} \tag{13}$$

where N_G, N_L, N_P are the numbers of DERs, loads and controller parameters subject to changes or disturbances; $\mathbf{F}_{\mathbf{x},G_i}$, $\mathbf{F}_{\mathbf{y},G_i}$, $\mathbf{G}_{\mathbf{x},G_i}$ are matrices only related to the fluctuations from the i^{th} DER; $\mathbf{F}_{\mathbf{x},L_j}$, $\mathbf{F}_{\mathbf{y},L_j}$, $\mathbf{G}_{\mathbf{x},L_j}$ are matrices only related to the changes of the j^{th} load; $\mathbf{F}_{\mathbf{x},P_k}$, $\mathbf{F}_{\mathbf{y},P_k}$, $\mathbf{G}_{\mathbf{x},P_k}$ are matrices only related to the perturbations from the k^{th} parameter; $\mathbf{F}_{\mathbf{x},E}$, $\mathbf{F}_{\mathbf{y},E}$, $\mathbf{G}_{\mathbf{x},E}$ are matrices only related to the disturbances at PCC; $\mathbf{F}_{\mathbf{x},C}$, $\mathbf{F}_{\mathbf{y},C}$, $\mathbf{G}_{\mathbf{x},C}$ are constant matrices uncorrelated with any disturbances. Based on (11)-(13), $\mathbf{A}_{MG,D}$ can be obtained as follows.

$$\begin{aligned}\mathbf{A}_{MG,D} &= \sum_{i=1}^{N_G}\mathbf{M}_{G_i} + \sum_{j=1}^{N_L}\mathbf{M}_{L_j} + \sum_{k=1}^{N_P}\mathbf{M}_{P_k} + \mathbf{M}_E \\ &+ \sum_{i=1}^{N_G}\sum_{j=1}^{N_L}\mathbf{M}_{G_i,L_j} + \sum_{i=1}^{N_G}\sum_{k=1}^{N_P}\mathbf{M}_{G_i,P_k} + \sum_{i=1}^{N_G}\mathbf{M}_{G,E} \\ &+ \sum_{j=1}^{N_L}\sum_{k=1}^{N_P}\mathbf{M}_{L_j,P_k} + \sum_{j=1}^{N_L}\mathbf{M}_{L_j,E} + \sum_{k=1}^{N_P}\mathbf{M}_{P_k,E} \end{aligned} \tag{14}$$

where $\mathbf{M}_{G_i}$, $\mathbf{M}_{L_j}$, $\mathbf{M}_{P_k}$, $\mathbf{M}_E$ represent the increments only caused by DERs, loads, controllers parameters, power exchange at PCC; the cross items $\mathbf{M}_{G_i,L_j}$, $\mathbf{M}_{G_i,P_k}$, $\mathbf{M}_{G_i,E}$, $\mathbf{M}_{L_j,P_k}$, $\mathbf{M}_{L_j,E}$, $\mathbf{M}_{P_k,E}$ represent their mutual effects on the matrix increment. Their expressions are given in Appendix A.

The advantage of the above matrix decomposition is that it becomes easy and efficient to calculate the increment $\mathbf{A}_{MG,D}$ when disturbances occur, because only specific sub-matrices need to be updated. Besides, it provides an efficient tool to analyze the impacts of disturbances. For instance, it can be clearly observed from (14) that the increment of state matrix can be expressed in the form of a combination of disturbances, which makes it easier to analyze the impact of a specific disturbance. Moreover, when a bounded set of uncertainties are introduced in system, it is easy to get the boundaries of sub-matrices first, and then to obtain $\mathbf{A}_{MG,D}$ correspondingly.

IV. Test Cases

A typical microgrid system shown in Fig. 1 is used to test and verify the presented approach. The test system includes three categories of DERs [7], namely non-dispatchable PV, dispatchable Fuel Cell (FC), and dispatchable Battery. Among these DERs, PV units are controlled via a maximum power point tracking strategy (P&O), whereas FC and Battery units are controlled in droop strategy [7]. Besides, the test system includes two types of loads [8], namely passive loads (constant impedance loads: Load2 and Load5 as shown in Fig. 1) and active loads (inverter-interfaced loads: Load1, Load3, and Load4 as shown in Fig. 1). The microgrid is assumed to operate in islanded mode in order to better illustrate the impact of disturbances on the small signal stability of a power-electronic-dominant system. Parameters for inverter controllers are summarized in Appendix B and the rest of microgrid parameters can be found in [9].

There are 132 eigenvalues in the test system. QR method is adopted first to calculate the eigenvalues and eigenvectors at the initial operation point [10]. Critical eigenvalues whose real parts are within the range $[-25, 1]$ are shown in Fig. 2.

As the microgrid is islanded, the disturbances of power exchange at PCC become zero. Further, sensitivity analysis

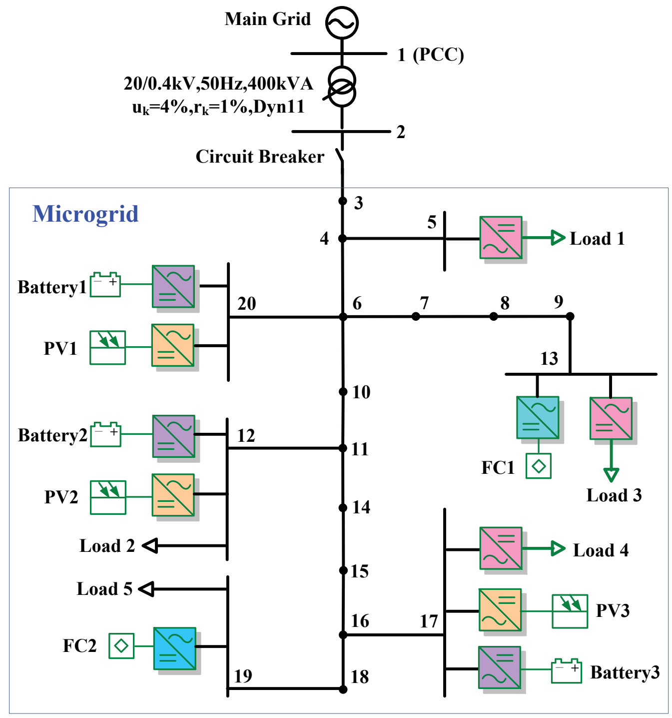

Fig. 1 Benchmark low-voltage microgrid network

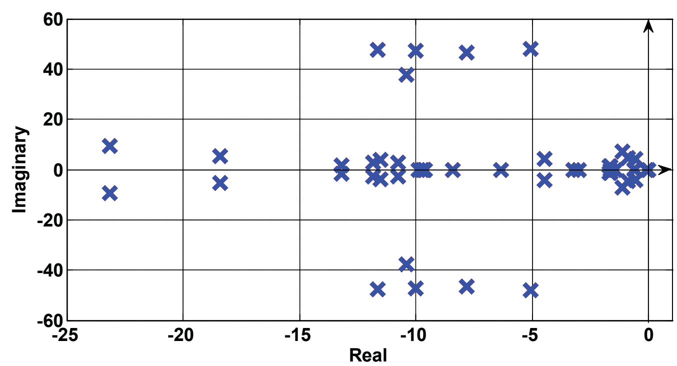

Fig. 2 Critical eigenvalues of the microgrid at initial state

shows that less stable eigenvalues are dominated by the power-electronic-interfaced DERs and loads including their controller parameters [8]. Thus eigen-analysis is focused on disturbances from these units.

A. Disturbances from DERs and Loads

The bounds for disturbances of PV irradiance and load power are shown in Table I. To better demonstrate the Geršgorin disk calculation, the initial values in Table I are set either as the lower bound or as the upper one. However, the Geršgorin approach is not limited to this boundary setting. Fig. 3 shows the comparison of Geršgorin disk calculations with and without quasi-diagonalization. Without quasi-diagonalization where $\mathbf{A}_{MG}$ is used directly for calculation, the calculated Geršgorin disk is too conservative to be useful (see the largest green circle in Fig. 3); on the contrary, tight

Table I Disturbances of PV Irradiance and Load Power

Disturbance No.	Object		Initial Value	Disturbance Bounds
1	Irradiance of PV1 (W/m^2)		1000.00	[970.00, 1000.00]
2	Irradiance of PV2 (W/m^2)		1000.00	[1000.00, 1050.00]
3	Irradiance of PV3 (W/m^2)		1000.00	[960.00, 1000.00]
4	Load1	Active Power (W)	12.75	[12.75, 14.00]
		Reactive Power (VAR)	7.90	[7.50, 7.90]
5	Load3	Active Power (W)	61.15	[61.15, 65.00]
		Reactive Power (VAR)	37.90	[36.00, 37.90]
6	Load4	Active Power (W)	12.75	[12.00, 12.75]
		Reactive Power (VAR)	7.90	[7.90, 8.50]

regions are obtained with quasi-diagonalization (see the red circles). This verifies the necessity and effectiveness of the quasi-diagonalization devised in Section III.

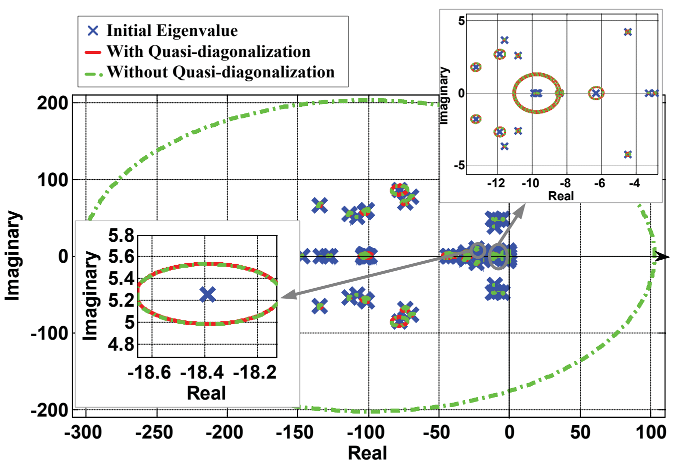

Fig. 3 Comparison of Geršgorin disks due to disturbance No. 1

Different Geršgorin disks of the critical eigenvalues under disturbances are shown in Fig. 4, which offers the following insights:

- Geršgorin theory provides a useful tool to analyze the impact levels of different disturbances. For instance, in this test, eigenvalue disks corresponding to the disturbance No. 4 are larger than the others, meaning this disturbance has a greater impact on the microgrid stability.
- Geršgorin theory can be combined with other methods to get a more accurate location for each eigenvalue [8]. For example, Fig. 4 shows that disks of seven eigenvalues $-8.4140+j0$, $-9.6505+j0$, $-9.6926+j0$, $-9.7305+j0$, $-9.7540+j0$, $-9.7540+j0$, and $-9.8989+j0$ are overlapped with each other. In this case, perturbation theory can then be utilized to focus on these eigenvalues calculation, instead of computing the entire system again when disturbances occur. As a demonstration, Table II gives the perturbation calculation results of these eigenvalues under the disturbances No. 1, No. 2, and No. 4. It verifies all eigenvalues are within their Geršgorin disks in Fig. 4.

B. Disturbances from Controller Parameters

We use Geršgorin theory to investigate the impact of droop coefficients on microgrid stability, because less stable modes in

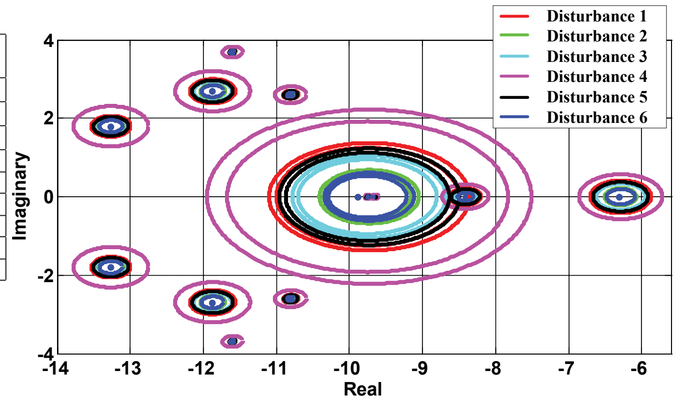

Fig. 4 Geršgorin disks due to disturbances shown in Table I

Table II Eigenvalues Calculation by Perturbation Theory

Initial Value	Disturbance No. 1	Disturbance No. 2	Disturbance No. 4
$-8.4140+j0$	$-8.4133+j0$	$-8.4141+j0$	$-8.4204+j0$
$-9.6505+j0$	$-9.6506+j0$	$-9.6499+j0$	$-9.6124+j0$
$-9.6926+j0$	$-9.6869+j0$	$-9.6867+j0$	$-9.6421+j0$
$-9.7305+j0$	$-9.7307+j0$	$-9.7303+j0$	$-9.7446+j0$
$-9.7540+j0$	$-9.7643+j0$	$-9.7605+j0$	$-9.7416+j0$
$-9.7540+j0$	$-9.7643+j0$	$-9.7605+j0$	$-9.7416+j0$
$-9.8989+j0$	$-9.8952+j0$	$-9.8960+j0$	$-9.9057+j0$

systems are influenced primarily by the droop coefficients. The uncertainty bounds of these coefficients are shown in Table III. Fig. 5 shows Geršgorin disk comparison between parameter disturbances and DERs (Loads) disturbances, providing the following insights:

- Droop coefficients have stronger impact on eigenvalues than disturbances in DERs and loads. It can be seen that Geršgorin disks due to DERs (Loads) disturbances are all covered by those due to parameter disturbances.
- Eigenvalues which are more sensitive to a specific parameter disturbance can be easily selected through Geršgorin disks analysis. For instance, the disk of the eigenvalue $-23.0368+j9.2424$ is larger than that of $-18.3862+j5.2594$, implying the former eigenvalue is more sensitive to this disturbance than the latter.
- Geršgorin disks due to parameter disturbances are more closely coupled with each other than those due to DERs (Loads) disturbances. For instance, the disks of eigenvalues whose real parts are within $[-18.3862, -4.9148]$ are overlapped with each other.
- Geršgorin theory also offers a tool to investigate the relationship between droop coefficients and critical modes, and such quantitative information can be further used to tune droop coefficients to better stabilize microgrid.

V. Conclusions

Uncertainties are widespread in microgrids. This problem becomes particularly challenging when heterogeneous unknown-but-bounded uncertainties exist in a microgrid. To tackle the challenge, an enhanced Geršgorin theory is presented for eigen-analysis of mcirogrids subject to uncertainties. With quasi-diagonalization, this method calculates Geršgorin

Table III Disturbances of Droop Coefficients

Unit	Parameter	Initial Value	Disturbance Bounds
Battery1	k_f	9.4001	$[9.4001, 9.6030]$
	k_v	10.3644	$[10.2541, 10.3644]$
Battery2	k_f	3.7136	$[3.5145, 3.7136]$
	k_v	8.0110	$[8.0110, 8.5105]$
Battery3	k_f	6.0346	$[6.0346, 6.3480]$
	k_v	4.9836	$[4.7789, 4.9836]$
FC1	k_f	0.4661	$[0.4661, 0.5657]$
	k_v	1.3947	$[1.3947, 1.7537]$
FC2	k_f	0.4061	$[0.4311, 0.4061]$
	k_v	6.1507	$[6.0593, 6.1507]$

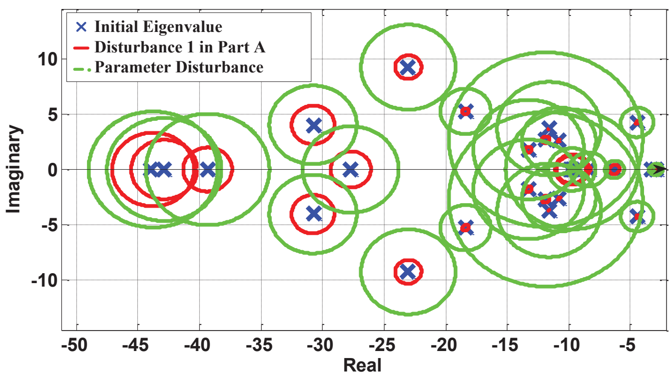

Fig. 5 Comparison of Geršgorin disks between parameter disturbances and DERs (Loads) disturbances

disks which give the distribution of microgrid eigenvalues under multiple disturbances. This robust stability analysis method is both mathematically rigorous and computationally efficient, offering a powerful tool for uncertainties analysis for microgrids. The Geršgorin based approach has been tested and verified via a typical microgrid with DERs.

APPENDIX

A. Sub-matrices Expression

$$\mathbf{M}_{G_i} = \mathbf{F}_{\mathbf{x},G_i} - \mathbf{F}_{\mathbf{y},G_i}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},G_i} - \mathbf{F}_{\mathbf{y},G_i}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},C} - \mathbf{F}_{\mathbf{y},C}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},G_i}$$
$$\mathbf{M}_{L_j} = \mathbf{F}_{\mathbf{x},L_j} - \mathbf{F}_{\mathbf{y},L_j}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},L_j} - \mathbf{F}_{\mathbf{y},L_j}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},C} - \mathbf{F}_{\mathbf{y},C}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},L_i}$$
$$\mathbf{M}_{P_k} = \mathbf{F}_{\mathbf{x},P_k} - \mathbf{F}_{\mathbf{y},P_k}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},P_k} - \mathbf{F}_{\mathbf{y},P_k}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},C} - \mathbf{F}_{\mathbf{y},C}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},P_k}$$
$$\mathbf{M}_{E} = \mathbf{F}_{\mathbf{x},E} - \mathbf{F}_{\mathbf{y},E}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},E} - \mathbf{F}_{\mathbf{y},E}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},C} - \mathbf{F}_{\mathbf{y},C}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},E}$$
$$\mathbf{M}_{G_i,L_j} = -\mathbf{F}_{\mathbf{y},G_i}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},L_j} - \mathbf{F}_{\mathbf{y},L_j}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},G_i}$$
$$\mathbf{M}_{G_i,P_k} = -\mathbf{F}_{\mathbf{y},G_i}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},P_k} - \mathbf{F}_{\mathbf{y},P_k}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},G_i}$$
$$\mathbf{M}_{G_i,E} = -\mathbf{F}_{\mathbf{y},G_i}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},E} - \mathbf{F}_{\mathbf{y},E}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},G_i}$$
$$\mathbf{M}_{L_j,P_k} = -\mathbf{F}_{\mathbf{y},L_j}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},P_k} - \mathbf{F}_{\mathbf{y},P_k}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},L_j}$$
$$\mathbf{M}_{L_j,E} = -\mathbf{F}_{\mathbf{y},L_j}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},E} - \mathbf{F}_{\mathbf{y},E}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},L_j}$$
$$\mathbf{M}_{P_k,E} = -\mathbf{F}_{\mathbf{y},P_k}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},E} - \mathbf{F}_{\mathbf{y},E}\mathbf{G}_{\mathbf{y}}^{-1}\mathbf{G}_{\mathbf{x},P_k}$$

B. Parameters for Inverter Controller

The controller of inverters adopted in this paper can be found in [7]. Controller parameters are given in Table IV.

Table IV Parameters for Inverter Controllers in the Microgrid

DERs (Loads)	Parameters					
Battery 1	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	0.40	0.02	0.40	0.02
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.008	0.10	0.02	0.10	0.02
Battery 2	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	0.45	0.02	0.45	0.02
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.008	0.12	0.02	0.12	0.02
Battery 3	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	0.50	0.02	0.50	0.02
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.008	0.13	0.02	0.13	0.02
PV 1	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	2.00	0.80	1.00	0.80
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.01	0.10	0.05	0.10	0.05
PV 2	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	1.50	0.80	1.00	0.80
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.01	0.10	0.06	0.10	0.06
PV 3	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	2.00	0.80	1.00	0.80
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.01	0.12	0.05	0.12	0.05
FC 1	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	0.80	0.01	0.80	0.01
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.008	0.10	0.02	0.10	0.02
FC 2	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	0.30	0.02	0.30	0.02
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.008	0.10	0.02	0.10	0.02
Load 1	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	0.20	0.05	0.20	0.05
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.008	0.80	0.02	0.05	0.05
Load 3	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	0.10	0.005	0.20	0.05
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.008	0.20	0.10	0.10	0.05
Load 4	Outer	T_f	K_P	T_P	K_Q	T_Q
	Loop	0.01	0.40	0.05	0.20	0.01
	Inner	T_m	K_d	T_d	K_q	T_q
	Loop	0.008	0.80	0.02	0.05	0.05

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